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Radiative Kinks and Fermionic Zero Modes in a $(\lambda\phi^6)_{1+1}$ Model

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We consider bidimensional scalar models including kink solutions $\phi_k(x)$. Using the hidden supersymmetric properties of the Dirac equation, we describe a general method to find normalizable fermionic zero modes. In particular, we apply the technique to a $(\lambda\phi^6)_{1+1}$ model. Going to the one-loop order of the effective potential, the emergence of a radiative kink provides an interesting scalar background in order to discuss the Dirac equation.

1. INTRODUCTION

Working on a topological background provided by the classical solutions of several theories (ranging from kinks or solitons in systems with one spatial dimension to monopoles in the three-dimensional ones), the fermionic number associated with the vacuum need not be an integer and can even result in a transcendental function of the parameters of the model (Goldstone and Wilczek, 1981). The so-called fractionization phenomenon can be analyzed according to different schemes. From a mathematical point of view it appears related to the η invariant of the Dirac Hamiltonian (Paranjape and Semenoff, 1983). Such objects, introduced in the analysis of the Index Theorem for noncompact manifolds, provide a regularized expression of the spectral asymmetry. In particular, we can find an interesting situation when the interaction term exhibits the charge conjugation symmetry C, thus making even the states with energy $+|E|$ with those others of $-|E|$. In these circumstances the physical interest of the problem, which normally spreads over all the spectrum, will only be concentrated on the zero-energy eigenstates. As a matter of fact, if these states are normalizable,

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each one of them adds minus one-half to the vacuum fermion number. To detect the zero-energy solutions of a model, the general tool to be used is the Index Theorem in open manifolds as first stated by Callias (1978) and Bott and Seeley (1978). In particular, the topological character of the phenomenon is easily understood by observing that the index depends on the behavior of the scalar background field at infinity. Moreover, an alternative study of the problem is feasible if we bear in mind the properties of the Dirac equation over a background $\phi_k(x)$. One starts from a pair of first-order equations which are easily decoupled, yielding a couple of Schrödinger-like equations. In fact, we find the conventional SUSY quantum mechanics situation where the fermion–boson interaction term $G(\phi_k)$ represents the superpotential $W(x)$ (Cooper *et al.*, 1988). In order to discuss the fermionic zero modes, we can exploit these hidden supersymmetric properties of the Dirac equation. First we recall the bosonic stability equation, where an unavoidable zero-energy mode emerges due to translational invariance. Bearing in mind the Schrödinger equations for the fermions, a clever choice for the coupling between scalar and Dirac fields percolates the bosonic stability equation to the fermionic part. In these conditions the emergence of a fermionic zero mode is guaranteed.

The existence of topological kinks or solitons requires an adequate vacuum degeneracy pattern. At tree order these objects appear as finite-energy solutions of the classical equations of motion. In several models the necessary symmetry breaking does not occur at the classical level. The phenomenon can come into play due to the quantum contributions, a dynamical symmetry breaking thus arising. Once the quantum corrections are taken into account, the unexpected vacuum degeneracy opens the possibility of topological kinks although the conventional tree-order ones were absent.

In order to discuss the behavior of a fermionic field over a specific bosonic background, we choose the theory of a scalar field in $(1+1)$ dimensions with self-interactions up to ϕ^6. The classical potential of the model exhibits three absolute minima (Lohe, 1979). Besides its applications in particle physics, the ϕ^6 self-interacting theory plays an important role in solid-state physics. In particular, it can be used to study the first-order phase transition from the ferroelectric to the paraelectric state and the structural phase transitions of crystals (Behera and Khare, 1980; Kittel, 1977). First we consider the tree-order kinks interpolating between adjacent vacua. Taking the Yukawa coupling, a nonnormalizable fermionic zero mode is found. A normalizable state is at hand if we apply the general technique that percolates the translational besonic zero mode to the fermionic part. Going to the one-loop order of the effective potential, we reach a physical picture very similar to the one outlined in models with dynamical symmetry breaking. In fact, the classical minimum located at $\langle\phi\rangle = 0$ disappears, so that only the nonzero minima survive. Therefore the vacuum of the model is twofold degenerate and the existence of a radiative topological kink with nonzero boundary values at infinity is feasible. Even maintaining the Yukawa coupling, this kink permits the existence of a normalizable fermionic zero mode.

The paper is arranged as follows. In Section 2 we discuss the general situation of the fermions over the tree-order kinks, while Section 3 deals with these problems in a $(\lambda\phi^6)_{1+1}$ model. Section 4 includes the one-loop order of the effective potential and the radiative kink emergence. Finally, we present our conclusions.

2. FERMIONS OVER TREE-ORDER KINKS

In this section we shall be concerned with a general model governed by the following Lagrangian density:

$$L = \tfrac{1}{2}(\partial_\mu\phi)^2 - V(\phi) + i\bar{\Psi}\gamma^\mu\partial_\mu\Psi - G(\phi)\bar{\Psi}\Psi \tag{2.1}$$

where ϕ corresponds to a real scalar field and Ψ represents a Dirac one. As usual, $V(\phi)$ is the well-behaved potential function. Since we are interested in models exhibiting kinks, the $V(\phi)$ potential should have at least two minima. In fact, the topological backgrounds will interpolate between two adjacent minima. In the sequel we consider the supersymmetric formulation for the $V(\phi)$ function, namely

$$V(\phi) = \tfrac{1}{2}U(\phi)^2 \tag{2.2}$$

which can always be achieved because $V(\phi)$ is essentially positive. With adequate normalization the minima of V will correspond to zeros of U, while the equations of motion are

$$\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} = -U(\phi)U'(\phi) \tag{2.3}$$

In particular, the energy functional can be written as

$$E[\phi] = \int \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 + \frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 + \frac{1}{2}\,U(\phi)^2\right] dx = \int \varepsilon(x)\,dx \tag{2.4}$$

which can always be achieved because $V(\phi)$ is essentially positive. With

$$\varepsilon(x) = \frac{1}{2}\left[\frac{d\phi}{dx} \mp U(\phi)\right]^2 \pm \frac{d\phi}{dx}\,U(\phi) \tag{2.5}$$

The Bogomolny inequality is simply

$$E[\phi] \geq \left|\int \frac{d\phi}{dx}\,U(\phi)\,dx\right| \tag{2.6}$$

with a saturated bound when we consider the kink solutions

$$\frac{d\phi}{dx} = \pm U(\phi) \tag{2.7}$$

so that we must solve a first-order equation. The constant solutions of (2.7) identify the tree-order vacua of the theory

$$\frac{d\phi_{0i}}{dx} = 0 \qquad \text{and} \qquad U(\phi_{0i}) = 0, \qquad i = 1, 2, \ldots, N \tag{2.8}$$

with a zero classical energy. The most general solution of (2.7) will include just an integration constant, namely

$$x - x_0 = \pm \int \frac{d\phi}{U(\phi)} \tag{2.9}$$

In the sequel we consider kinks centered at $x_0 = 0$. As a matter of fact, when $U(\phi)$ has a unique absolute minimum there can be no kinks, while if $U(\phi)$ exhibits n discrete degenerate minima we find $2(n-1)$ types of tree-order kinks. Moreover, each one of them will interpolate between any two neighboring minima as x varies from $-\infty$ to $+\infty$. In principle, a first approach to the global quantum theory over the inhomogeneous classical solutions is obtained by taking the small perturbations over them. If $\phi_k(x)$ represents the background, once we expand the quantum field $\phi(x, t)$ in the form

$$\phi(x, t) = \phi_k(x) + \varphi(x, t) \qquad \text{with} \quad \varphi(x, t) = \sum_j \varphi_j(x) \exp(i\omega_j t) \tag{2.10}$$

we can obtain the stability equation

$$-\frac{d^2\varphi_j}{dx^2} + V''(\phi_k)\varphi_j = \omega_j^2 \varphi_j \tag{2.11}$$

where

$$V''(\phi_k) = U'(\phi_k)^2 + U(\phi_k)U''(\phi_k) \tag{2.12}$$

In particular, the mandatory $\omega_j = 0$ solution can be easily understood: it represents the bosonic zero mode due to translational invariance (Rajaraman, 1982). Returning to the fermionic part, once we choose as representation for the two-dimensional γ matrices $\gamma_0 = \sigma_2$ and $\gamma_1 = i\sigma_3$, the fluctuations built over the kink are given by

$$[i\gamma^\mu \partial_\mu - G(\phi_k)]\Psi = 0 \tag{2.13}$$

If we write the spinor in its two-component form

$$\Psi(x, t) = \begin{bmatrix} u(x) \\ v(x) \end{bmatrix} \exp(i\omega_F t) \tag{2.14}$$

then equation (2.13) transforms into

$$i\left[\frac{d}{dx}+G(\phi_k)\right]v(x)=-\omega_F u(x) \tag{2.15a}$$

$$i\left[\frac{d}{dx}-G(\phi_k)\right]u(x)=-\omega_F v(x) \tag{2.15b}$$

In order to detect the zero-energy solutions of (2.13), the mathematical tool to be applied is the Index Theorem in open spaces, as first stated by Callias, Bott, and Seeley. Taking the z-regularization technique, the index would read (Callias, 1978; Bott and Seeley, 1978)

$$\Delta(z)=\frac{1}{2}\left[\frac{G_+}{(G_+^2+z)^{1/2}}-\frac{G_-}{(G_-^2+z)^{1/2}}\right] \tag{2.16}$$

on the understanding that $G_\pm$ represent the values of $G(\phi_k)$ as x goes to infinity. Following the conventional analysis, the number of zero-energy states is obtained through the $z\to 0$ limit

$$\Delta(z\to 0)=\tfrac{1}{2}\,\mathrm{sign}(G_+)-\tfrac{1}{2}\,\mathrm{sign}(G_-) \tag{2.17}$$

so that whenever $G_+>0>G_-$, $\Delta(z\to 0)=1$, and therefore we just find a normalizable zero mode. In these conditions we can state the requirement for a fermionic zero-energy mode: it needs an interaction term that, once considered, the kink $\phi_k(x)$ takes at infinity values fulfilling $G_+>0>G_-$.

However, an alternative analysis of the problem is possible if we bear in mind the hidden SUSY quantum mechanics character of the Dirac equation over the background $G(\phi_k)$. As a matter of fact, we can decouple (2.15),

$$\left[-\frac{d^2}{dx^2}+G(\phi_k)^2+\frac{dG(\phi_k)}{dx}\right]u(x)=\omega_F^2 u(x) \tag{2.18a}$$

$$\left[-\frac{d^2}{dx^2}+G(\phi_k)^2-\frac{dG(\phi_k)}{dx}\right]v(x)=\omega_F^2 v(x) \tag{2.18b}$$

thus obtaining the pair of Schrödinger equations associated with a SUSY quantum mechanics exercise [the $G(\phi_k)$ function represents the superpotential $W(x)$]. We can recall the conventional formulation of these models (Witten, 1982)

$$H=S^2,\qquad S=\begin{bmatrix}0 & Q^+\\ Q & 0\end{bmatrix} \tag{2.19}$$

with

$$Q=\left[-\frac{d}{dx}+W\right],\qquad Q^+=\left[\frac{d}{dx}+W\right] \tag{2.20}$$

In fact, the supersymmetric properties of the theory can be concentrated into the transformation

$$S\begin{bmatrix} u \\ v \end{bmatrix} = \sqrt{E}\begin{bmatrix} u \\ v \end{bmatrix} \tag{2.21}$$

Using (2.19), the Hamiltonian operator is simply

$$\begin{bmatrix} H_- & 0 \\ 0 & H_+ \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} = E\begin{bmatrix} u \\ v \end{bmatrix} \tag{2.22}$$

which in a more transparent version reads

$$\left[-\frac{d^2}{dx^2} + W^2 + \frac{dW}{dx}\right] u = Eu \tag{2.23a}$$

$$\left[-\frac{d^2}{dx^2} + W^2 - \frac{dW}{dx}\right] v = Ev \tag{2.23b}$$

The SUSY quantum mechanics represents an adequate frame within which to discuss the supersymmetry spontaneous breaking patterns. As SUSY remains unbroken only if the energy of the ground state is zero, the physical interest of the problem concentrates on the zero modes. The situation can be analyzed using the so-called Witten index, an order parameter which in several cases provides valuable information about the symmetry-breaking phenomenon. Maintaining the β-regularization of Witten (1982), the mentioned object adopts the form

$$\Delta(\beta) = \mathrm{Tr}[\exp(-\beta H_+) - \exp(-\beta H_-)] \tag{2.24}$$

It represents a measurement of the difference of $u(x)$ and $v(x)$ eigenstates with zero energy (let us point out that for positive energies duplication occurs between both parts). In principle, the index cannot depend on the regularization parameter β. However, the explicit computations sometimes end up in a Witten index with a β dependence (even taking the limit $\beta \to \infty$, cumbersome final results such as 1/2 are obtained). To sum up, we can point to the form taken by $\Delta(\beta)$ in terms of the diagonal parts of the heat kernels associated with H_+ and H_- (Akhoury and Comtet, 1984)

$$\Delta(\beta) = \int [K_+(x, x, \beta) - K_-(x, x, \beta)]\, dx \tag{2.25}$$

while the equations satisfied by the heat kernels are

$$\left[\frac{\partial}{\partial\beta} - \frac{\partial^2}{\partial x^2} + W^2 \mp \frac{dW}{dx}\right] K_\pm = 0 \tag{2.26}$$

If we make a good use of the following result, which appears in Niemi and Semenoff (1986),

$$\frac{d}{d\beta}[K_+(x,x,\beta)-K_-(x,x,\beta)]=\frac{1}{2}\frac{d}{dx}\left[\frac{d}{dx}+2W\right]K_+(x,x,\beta) \quad (2.27)$$

we can reach the final result

$$\frac{d\Delta(\beta)}{d\beta}=\frac{1}{(4\pi\beta)^{1/2}}[W_+\exp(-\beta W_+^2)-W_-\exp(-\beta W_-^2)] \quad (2.28)$$

Since $W_+>0$ and $W_-<0$, we perform the integration with $\Delta(0)=0$

$$\Delta(\beta)=\tfrac{1}{2}\Phi(W_+\sqrt{\beta})+\tfrac{1}{2}\Phi(W_-\sqrt{\beta}) \quad (2.29)$$

where Φ represents the probability Fresnel function. Taking the limit $\beta\to\infty$ (we recall the $z\to 0$ limit for the conventional Index Theorem), the Witten index amounts to 1. Cumbersome situations can arise if either W_+ or W_- is zero. For example, with $W_-=0$ we get

$$\Delta(\beta)=\tfrac{1}{2}\Phi(W_+\sqrt{\beta}) \quad (2.30)$$

so that the limit $\beta\to\infty$ leads to $\Delta=1/2$, an identical value to the one obtained within the Liouville SUSY quantum mechanics (Niemi and Wijewardhana, 1984). In the bosonic sector of this model the energy going to the zero limit of the continuum normalized wave function yields a non-square-integrable state that tends to zero as $x\to\infty$, while it reaches a nonzero value if $x\to-\infty$. In these conditions the emergence of a normalizable fermionic zero mode requires nonzero superpotential at infinity.

We can otherwise exploit the hidden supersymmetric properties of the Dirac equation over the scalar background provided by $\phi_k(x)$. It suffices to consider the bosonic stability equation (2.11) with its zero-energy eigenstate due to translational invariance. Making a clever choice for the $G(\phi)$ function, we can percolate the scalar stability equation (zero mode included) to the fermionic part. In particular, $G(\phi)=gU'(\phi)$ (g represents a dimensionless constant) is the right bosonic–fermionic coupling. According now to the specific sign of the Bogomolny condition (2.7) and with the new spatial coordinate $y=gx$, the bosonic equation (2.11) coincides either with (2.18a) or with (2.18b). Therefore the zero-energy eigenmode is a common characteristic for both bosonic and fermionic sectors. Furthermore, the fermionic zero mode can be determined using first-order differential equations [see equations (2.20) and (2.21)]. The former arguments permit us, for example, to recover the conventional Yukawa boson–fermion coupling in field theories with ϕ^4 self-interaction (Jackiw and Rebbi, 1976). In the sequel we shall go to a bidimensional scalar field with self-interactions up to ϕ^6.

3. THE $(\lambda\phi^6)_{1+1}$ MODEL

The model we shall be concerned with corresponds to a real self-interacting scalar field in (1+1) dimensions governed by the Lagrangian density (Lohe, 1979)

$$L=\tfrac{1}{2}(\partial_\mu\phi)^2-\tfrac{1}{2}\lambda^2\phi^2\left(\phi^2-\frac{m}{\lambda}\right)^2 \tag{3.1}$$

where m and λ are positive constants with mass dimensionality. The classical potential has three absolute minima: one at $\langle\phi\rangle=0$ and the others at $\langle\phi\rangle=\pm(m/\lambda)^{1/2}$ (Figure 1). At this tree level we can point out the existence of a kink as (Lohe, 1979)

$$\phi_{\rm k}(x)=\left[\frac{m}{2\lambda}(1+\tanh mx)\right]^{1/2} \tag{3.2}$$

a field configuration which makes a smooth interpolation between the vacua $\phi=0$, $\phi=(m/\lambda)^{1/2}$ (Figure 2). More solutions are also possible by putting $x\to -x$ and $\phi_k\to-\phi_k$. In particular, we will employ the antikink

$$\phi_{\rm ak}(x)=\left[\frac{m}{2\lambda}(1-\tanh mx)\right]^{1/2} \tag{3.3}$$

now with an interpolation between $\phi=(m/\lambda)^{1/2}$ and $\phi=0$. As the tree-order kinks only connect two neighboring minima as x varies from $-\infty$ to ∞, we cannot find a classical solution interpolating between the vacua $\pm(m/\lambda)^{1/2}$. With this set of minima the system admits four (anti) kinks.

In principle we consider the $(\lambda\phi^6)_{1+1}$ theory including a Yukawa coupling to Dirac fermions, a model then governed by the Lagrangian

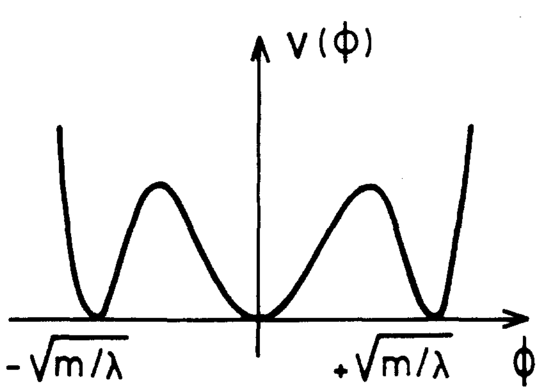

Fig. 1. The classical potential of the ϕ^6 model.

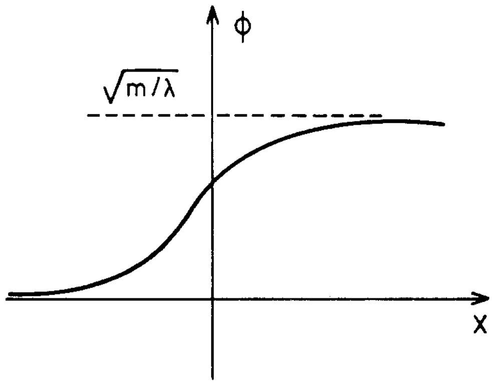

Fig. 2. The "tree-order" kink interpolating smoothly between the vacua $\phi=0$, $\phi=(m/\lambda)^{1/2}$.

density

$$L=\tfrac{1}{2}(\partial_\mu\phi)^2-\tfrac{1}{2}\lambda^2\phi^2\left(\phi^2-\frac{m}{\lambda}\right)^2+i\bar{\Psi}\gamma^\mu\partial_\mu\Psi-f\phi\bar{\Psi}\Psi \tag{3.4}$$

(f is the fermionic coupling constant with mass dimensionality). Working over the background provided by the kink of (3.2), equations (2.18) transform into

$$\left[-\frac{d^2}{dx^2}+f^2\phi_k^2+f\frac{d\phi_k}{dx}\right]u(x)=\omega_F^2u(x) \tag{3.5a}$$

$$\left[-\frac{d^2}{dx^2}+f^2\phi_k^2-f\frac{d\phi_k}{dx}\right]v(x)=\omega_F^2v(x) \tag{3.5b}$$

while the hypothetical zero modes can be determined through the first-order differential equations

$$\left[\frac{d}{dx}+f\phi_k\right]v=0 \tag{3.6a}$$

$$\left[-\frac{d}{dx}+f\phi_k\right]u=0 \tag{3.6b}$$

As regards the SUSY quantum mechanics pattern, we have a superpotential as

$$W(x)=\left[\frac{f^2m}{2\lambda}(1+\tanh mx)\right]^{1/2} \tag{3.7}$$

Since W_- is zero, we finally obtain a Witten index equal to 1/2. This embarrassing result can be understood by returning to (3.6). As a matter of fact, the hypothetical zero mode behaves as

$$v(x) \to 0 \qquad \text{as} \quad x \to \infty \tag{3.8a}$$

$$v(x) \to \left[\frac{\sqrt{2}+1}{\sqrt{2}-1}\right]^{f/(2m\lambda)^{1/2}} \qquad \text{as} \quad x \to -\infty \tag{3.8b}$$

while the $u(x)$ solution diverges as $x \to \infty$. Taking the antikink solution, it is $u(x)$, which exhibits the behavior of (3.8). A normalizable fermionic zero mode is feasible by applying the general technique exposed in Section 2. We are now concerned with

$$L = \tfrac{1}{2}(\partial_\mu \phi)^2 - \tfrac{1}{2}\lambda^2 \phi^2 \left(\phi^2 - \frac{m}{\lambda}\right)^2 + i\bar{\Psi}\gamma^\mu \partial_\mu \Psi - f\left(3\phi^2 - \frac{m}{\lambda}\right)\bar{\Psi}\Psi \tag{3.9}$$

Taking again the kink solution of (3.2), we find a superpotential as

$$W(x) = \frac{fm}{2\lambda}(3 \tanh mx + 1) \tag{3.10}$$

so that W_+ and W_- will be different from zero. In these conditions the Witten index finally amounts to 1. The existence of a square-integrable fermionic state with zero energy can be easily found using (3.6). We find

$$\Psi_0(x) = v_0 \begin{bmatrix} 0 \\ (\cosh mx)^{-3f/2\lambda} \exp(-fmx/2\lambda) \end{bmatrix} \tag{3.11}$$

where v_0 is the normalization constant. The antikink solution leads to $\Delta = -1$ with a nonzero upper component for the zero mode.

4. RADIATIVE KINKS AND FERMIONIC ZERO MODES IN $(\lambda\phi^6)_{1+1}$

The existence of topological kinks requires an adequate vacuum degeneracy pattern. Up till now we have analyzed the tree-order kinks that appear as finite-energy solutions of the classical equations of motion once the right boundary conditions are imposed. We can find several models where, although the spontaneous symmetry breaking does not appear at the classical level, it comes into play when considering the quantum corrections. In these conditions we face a dynamical spontaneous symmetry breaking (Coleman and Weinbers, 1973), so that the radiative vacuum degeneracy permits the existence of topological kinks even though the classical ones were absent. Returning to our $(\lambda\phi^6)_{1+1}$ model of (3.1), we dispose of a Lagrangian exhibiting the $\phi \to -\phi$ internal discrete symmetry.

As the classical potential has three absolute minima, we find an embarrassing situation: the vacua around $\langle\phi\rangle = \pm(m/\lambda)^{1/2}$ would represent the spontaneous symmetry-breaking phenomenon, while if the system chooses the $\langle\phi\rangle = 0$ minimum, the symmetry maintains its exact character. Fortunately, we can remove the problem only going to the one-loop order of the effective potential (Babu Joseph and Kuriakose, 1982). While the zero-loop or tree-order contribution to the effective potential is simply

$$V_0(\phi) = \tfrac{1}{2}\lambda^2\phi^2\left(\phi^2 - \frac{m}{\lambda}\right)^2 \tag{4.1}$$

the one-loop correction is computed through the Gaussian approximation. In our case

$$V_1(\phi) = \frac{1}{2}\int \frac{d^2k}{(2\pi)^2}\ln(k^2 + M^2) \tag{4.2}$$

where

$$M^2 = m^2 - 12\lambda m\phi^2 + 15\lambda^2\phi^4 \tag{4.3}$$

Now we can pass over the ultraviolet divergent character of (4.2) using the conventional cutoff parameter. Afterward we impose the following normalization conditions (Babu Joseph and Kuriakose, 1982):

$$V(\phi = (m/\lambda)^{1/2}) = 0 \tag{4.4a}$$

$$\frac{d^2V(\phi = (m/\lambda)^{1/2})}{d\phi^2} = 4m^2 \tag{4.4b}$$

$$\frac{d^4V(\phi = (m/\lambda)^{1/2})}{d\phi^4} = 156\lambda m \tag{4.4c}$$

so that the renormalized one-loop effective potential reads

$$V(\phi) = \tfrac{1}{2}\lambda^2\phi^2\left(\phi^2 - \frac{m}{\lambda}\right)^2 + \frac{M^2}{8\pi}\ln\left(\frac{4m^2}{M^2}\right) \tag{4.5}$$

In this way the minimum $\langle\phi\rangle = 0$ disappears and the vacua around any one absolute minimum $\pm(m/\lambda)^{1/2}$ would correspond to spontaneous symmetry breaking. Therefore, we find a model where the vacuum degeneracy drawn at one-loop order differs from the one outlined using the tree order. With a vacuum twofold degenerate, the existence of a radiative topological kink $\phi_{\mathrm{kr}}(x)$ is possible (Figure 3). We only need the boundary conditions

$$\lim_{x\to\pm\infty} \phi_{\mathrm{kr}}(x) = \pm(m/\lambda)^{1/2} \tag{4.6}$$

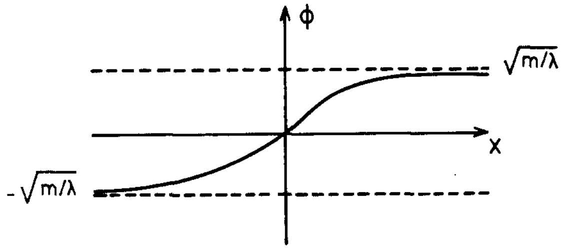

Fig. 3. A kink configuration based on the "one-loop" order.

This situation resembles the Gross–Neveu model with a radiative symmetry breaking for the condensate field σ (Dashen *et al.*, 1975). The space-dependent field configurations $\sigma(x)$ lead the phase transition at finite temperature. Returning to $(\lambda\phi^6)_{1+1}$, we must review the fermionic problem once the vacuum degeneracy has changed due to quantum corrections. Maintaining the Yukawa coupling of (3.4), we consider the radiative topological kink fulfilling (4.6). In these conditions the boundary values of the superpotential $W(x)$ are simply

$$W_- = -f(m/\lambda)^{1/2}, \qquad W_+ = f(m/\lambda)^{1/2} \tag{4.7}$$

so that the Witten index amounts to 1 and our topological arguments lead us to conclude the existence of a square-integrable fermionic zero mode over the background provided by the kink $\phi_{\mathrm{kr}}(x)$. Furthermore, we can recover the conventional fractionization phenomenon (see Section 1).

5. CONCLUSIONS

We considered bidimensional scalar models with an adequate vacuum degeneracy pattern in order to find tree-order topological kinks $\phi_{\mathrm{k}}(x)$. Adding fermions, the hidden supersymmetric properties of the Dirac equation yield a general method to find square-integrable fermionic zero modes. We applied the technique to a $(\lambda\phi^6)_{1+1}$ theory with three absolute minima at tree order. Going to the one-loop contribution to the effective potential, the vacuum degeneracy changes, a situation very similar to the one outlined in models with dynamical symmetry breaking. We take the radiative kink interpolating between the vacua $\pm(m/\lambda)^{1/2}$. Finally, topological arguments lead us to conclude the existence of a square-integrable fermionic zero mode even maintaining the Yukawa boson–fermion interaction.

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